

THE SIGNATURE OF THE NEGATIVE CURVATURE OF THE UNIVERSE IN CMB MAPS

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Abstract The geodesics followed by cosmic microwave background (CMB) photons show different behaviours depending on the geometry of space. Namely, the effect of ‘mixing geodesics’ predicts a distinct signature in CMB maps: threshold-independent elongated anisotropy spots in negatively curved geometries. We have found statistically significant sign for spot elongation in the COBE four year maps. This can be a direct indication for the negative curvature of the Universe.

1 The Predicted Feature of CMB Maps

It appears that the evidence in favor of a negatively curved Universe is rapidly gaining weight [1]. Here we present the results of a statistically independent analysis of the four year COBE-DMR data which favors a Friedmannian universe with $k = -1$ and hence, $\Omega < 1$ [2]. Our analysis is based on the search of the signature of the effect of *geodesic mixing* in these data.

The study of the projection of geodesics from (3+1)-dimensional Lorentzian space to a 3D Riemannian one, and of the behavior of time correlation functions for geodesic flows on homogeneous isotropic spaces with negative curvature leads to the effect of “geodesic mixing” [3, 4] (and references therein). Geodesic flows, being Anosov systems (locally if the space is not compact), are exponentially unstable systems possessing the strongest statistical properties (mixing) and positive Kolmogorov-Sinai (KS) entropy h . For a geodesic flow in $k = -1$ Friedmannian Universe the KS-entropy depends on the only parameter a , the diameter of the Universe: $h = 2/a$ (Lyapunov exponents vanish when $k = 0, +1$).

The geodesic mixing has the following observable consequences: (1) decrease of the amplitude of CMB anisotropy by time, namely, the anisotropy detected now should be lower than at the surface of last scattering; (2) flattening of CMB angular autocorrelation function independent on the initial spectrum at the last scattering epoch (however, higher anisotropy at

measurements with smaller beam angles is also predicted by mixing); (3) threshold independent distortion of CMB maps. Concerning the third effect, the relation between the quantitative measurement of the distortion of patterns - spot elongation parameter, ϵ , and Ω is given by [4]

$$\frac{\ln(1/\epsilon)}{(\frac{1-\Omega_0}{1+z\Omega_0})^{\frac{1}{2}}} = \begin{cases} \alpha/(1-\alpha)[1-(1+z_1)^{1-1/\alpha}] & \alpha < 1 \\ \ln(1+z_1) & \alpha = 1 \end{cases} \quad (1)$$

where Ω_0 is its present value, z_1 corresponds to the time when matter becomes non relativistic and z to the decoupling time, α is the power index in the expansion law of the Universe. Note that:

(a) The first two effects - the decrease of anisotropy and the flattening of the autocorrelation function - are consequences of the negative curvature, while effect (3) on the map distortion, is a feature only of perturbed $k = -1$ Universe.

(b) Einstein equations do contain information on the curvature but not on the topology of the Universe; for example, the same $k = 0$ curvature can correspond to various topologies - R^3 , Tor^3 , $R^2 \times S^1$, $R^1 \times Tor^2$, etc;

(c) the use of harmonic analysis, being the main tool for study of CMB properties for $k = 0$ models, has intrinsic difficulties in hyperbolic spaces, due to problems with incompleteness of the set of eigenfunctions; moreover, their spectrum can be continuous or even not defined at all depending on the topology (Sobolev problem).

The use of methods of the theory of dynamical systems enables one to avoid in a way the principal difficulty of harmonic analysis. The followed geodesic mixing as statistical effect is a result of the photon beam motion from the last scattering epoch up to the present observer, independent of the properties of the peaks of anisotropies at the last scattering surface [5]. Thus, the effect we are looking brings information on the curvature but not on the topology of Universe.

2 Data Analysis: Checking the Prediction

In order to characterize the presence of elongated hot spots in CMB maps we have adopted a strategy that relies on the statistics of topological descriptors, which has been successfully used to place important restrictions on the spectrum of primordial perturbations $P(k)$ [6, 8], The analysis presented here is based on the 4 year COBE-DMR maps at 53 GHz [9]. Signal and noise maps were prepared by adding and subtracting the two independent

Table 1: Eccentricity parameter of hot spots on COBE maps (ϵ_ν^{A+B} , ϵ_ν^{A-B}) and comparison with Monte Carlo noise maps (ϵ_ν^{MC}). Δ^{A+B} and Δ^{A-B} denote the difference (in standard deviations) between the measured eccentricities and the mean eccentricity of noise Monte Carlo maps.

ν	ϵ_ν^{A+B}	ϵ_ν^{A-B}	ϵ_ν^{MC}	Δ^{A+B}	Δ^{A-B}
1.00	0.480	0.570	0.582	3.551	0.447
1.25	0.547	0.590	0.617	2.271	0.886
1.50	0.497	0.616	0.646	4.288	0.881
1.75	0.576	0.691	0.674	2.432	-0.429
2.00	0.600	0.686	0.699	2.049	0.267
2.25	0.595	0.819	0.719	2.060	-1.675
2.50	0.427	0.685	0.741	4.017	0.719
2.75	0.574	0.732	0.761	1.752	0.272
3.00	0.507	0.705	0.776	1.815	0.480

DMR channels (i.e. $0.5A + 0.5B$ and $0.5A - 0.5B$) and Gaussian smoothing ($\sigma = 2.9^\circ$). The geometric characteristics of hot spots are quite sensitive to galactic cuts below 15° to 20° but beyond 20° our results are stable. A hot spot with preset temperature threshold $T_\nu = \nu\sigma$ is defined, where σ is the standard deviation of the sky temperature. The algorithm of the topological analysis of CMB maps [6, 7], the ‘eccentricity parameter’, ϵ_ν , as the average ratio of the shortest to longest ‘axis’ of a hot spot for different temperature thresholds. In order to evaluate the statistical significance of the result we have performed Monte Carlo studies of noise maps that take into account instrumental noise and COBE’s beam width; the same algorithm was used for them (Table 1). The χ^2 statistic computed with the 9 data points in the range $\nu = 1.0 - 3.0$ and the corresponding noise Monte Carlo points is 5.6 and 73.0 for the $(A - B)$ and $(A + B)$ maps respectively, thus indicating the accuracy of the Monte Carlo simulations. On the other hand, the high χ^2 obtained when the data from the signal maps is compared with noise data is a clear indication of an actual detection of elongated anisotropy spots. The average deviation in terms of standard deviations of ϵ_ν^{A+B} from the

corresponding Monte Carlo result for the 9 bins considered here is 3σ .

Thus, the excess elongation as possible genuine feature of the hot spots on COBE maps is well established at least at threshold levels ν between 1.0 and 3.0. The detected signal shows a statistically strong independence on the threshold, contrary to the case for both $(A - B)$ and Monte Carlo maps, where ϵ_ν shows a clear correlation with threshold. Fluctuations on the surface of last scattering might also produce non-zero ellipticity even in flat universe [10], which however will depend on the threshold, contrary to what we have observed. If the effect of elongation detected on CMB maps is due to the geodesic mixing, as predicted, then our analysis implies the negative curvature of the Universe.

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